

Sample for review....

Calculus IIIHamdan, Fall 2011

1. Integrate the following

(a) $\int \frac{x dx}{4 + 9x^4}$

(b) $\int \cos^{-1} x dx$

2. Simplify

(a) $\cos\left(\tan^{-1}\left(\frac{5}{\sqrt{x}}\right)\right)$.

(b) $10 \cosh(\ln x)$

Evaluate the following (improper) integrals:

(a) $\int_0^{\infty} \frac{2 dx}{4 + x^2}$

(b) $\int_8^{\infty} \frac{dx}{(x-7)(x-6)}$

3. Determine whether the following improper integrals converge or diverge:

(a) $\int_1^{\infty} \frac{\sqrt{x+2}}{x^5} dx$

(b) $\int_0^3 \frac{dx}{\sqrt{3-x}}$

(c) $\int_0^{\infty} \frac{\sqrt{x}}{1+x^2} dx$

(d) $\int_1^{\infty} \frac{|\ln x|}{x} dx$

(e) $\int_1^{\infty} \frac{\ln x}{(x + \sqrt{x-1})} dx$

(f) $\int_1^{\infty} \frac{e^x}{\sqrt{1+x^2}} dx$

(g) $\int_0^{\infty} \frac{|\sin x|}{1+x^2} dx$

4. Find the value of $p > 0$ for which the integral $\int_2^{\infty} \frac{1}{x(\ln x)^p} dx$ converges.

5. Determine whether the improper integrals converge or diverge:

(a) $\int_{-\infty}^{-5} \frac{8}{x^3} dx$

(b) $\int_1^{\infty} \frac{e^x}{\sqrt{1+x^2}} dx$

6. Determine whether the following series converges or diverges. Name the test you are using:

(a) $\int_1^{\infty} \frac{2x^2 - 3x + \ln x}{2x - 1} dx$

(b) $\int_1^{\infty} \frac{e^{-x}}{x^3} dx$

(c) $\int_1^{\infty} \frac{\ln x}{x^{1.5}} dx$

(d) $\int_0^1 \frac{e^{-x}}{x} dx$

(d) $\int_0^\pi \frac{dx}{x + \sin x}$

9. Sort from slowest to fastest the following functions: No need to justify:

$$x^e, \quad e^x, \quad (\ln x)^x, \quad \ln(x^x), \quad \ln(x^2)$$

Consider the improper integral $\int_1^\infty \frac{dx}{(x^2 + 1)^3}$.

(a) Determine whether it converges or diverges, **using the comparison test**.

(b) What can you say about $\int_0^\infty \frac{dx}{(x^2 + 1)^3}$?